Appendix F

PY205M: Modern Mechanics Fluids Supplement, V1.0

Karen Daniels, NC State University
April 11, 2009

F.1 What is a Fluid?

So far, we have focused on understanding the mechanics of solid objects (including the microscopic origins of the Young's modulus, speed of sound, and specific heat) through the Momentum and Energy Principles as applied to a ball-and-spring model. These same two fundamental principles can also describe the mechanics of fluids. As with other topics this semester, we will make use of both microscopic and macroscopic reasoning to understand the statics and dynamics of fluid systems.

A "fluid" is generally meant to include both gases and liquids, with the key distinguishing feature being whether the fluid is compressible (gas) or not (liquid). We observe that a gas will expand to fill a closed container, while a fluid will simply settle to the bottom. The mechanism behind this ability to fill a container lies in whether the thermal energy of the molecules is enough to destroy all connection to their neighbors. Thus, by adding thermal energy to a system, you can cause a phase transition from liquid to gas. (In fact, adding thermal energy to a system will also cause a phase transition from solid to liquid.)

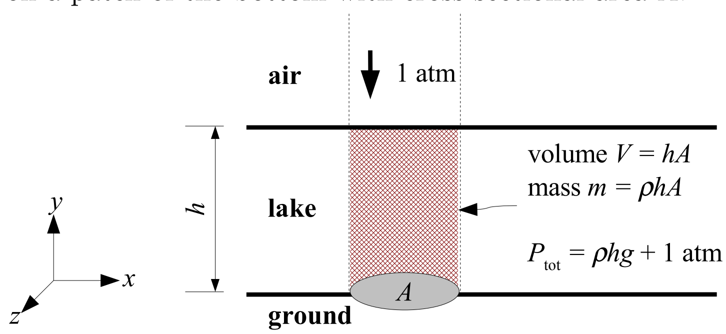
Fluids can be distinguished from solids in one key macroscopic way: the cannot hold their own shape. This results from a specific microscopic difference: due to having enough thermal energy to break bonds with their neighbors, the molecules of a fluid are constantly rearranging among themselves. Thus, there is no crystalline structure of the sort we used for the ball-and-spring model of a solid. In general, it is also true solids have more closely-packed molecules, and therefore a higher density than the liquid phase of the same material. (An interesting exception to this is ice.)

We can model the microscopic behavior of a fluid as a multi-particle system of molecules moving and colliding randomly, or as a macroscopic object with a mass and a center-of-mass velocity. In the microscopic view, the particles have thermal energy, and interact with each other through intermolecular forces which typically have short-range repulsion (due to overlapping electron orbitals) and long-range attraction (due to something called the van der Waals force). This contrasts with the ball-and-spring model of a solid, where the atoms were at fixed locations in a crystal. The shape of a fluid is typically determined by the shape of its container, and for liquids there is a *surface tension* which holds together the molecules along any unbounded surface. Macroscopically, we observe that a fluid can readily rearrange its shape in response to external forces.

F.2 Fluid Statics: Pressure

F.2.1 Superposition of pressures

Consider a the ground at the bottom of a lake. We want to calculate the force that the water and the air above exert on a patch of the bottom with cross-sectional area A:



First, we will consider the system macroscopically, using the superposition of forces which are pushing down on the patch. Start by considering the "block" of water and the "block" of air which are in a column above the patch of ground: they are pushing down, and the ground is pushing back up with an equal and opposite force (since we observe no change in momentum, $\Delta\vec{p}_{water} = 0$). We can calculate the mass of each "block" (water and air) and determine the force they exert in the $-\hat{y}$-direction:

$$F_y^{water} = -m_{water}g = -(\rho_{water}V)g = -(\rho_{water}Ah_{water})g$$
$$F_y^{air} = -m_{air}g = -(\rho_{air}V)g = -(\rho_{air}Ah_{air})g$$

Where we have taken advantage of the fact that we know the volume V of the block is the area A multiplied by its height h. We can convert these expressions to be written as pressures instead of forces. Recall from our discussion of Young's modulus that pressure is a force per unit area, and that we're talking about the same area A in both of the above expressions.

$$P_{water} = \frac{|F_{water}|}{A} = \rho_{water}gh_{water}$$
$$P_{air} = \frac{|F_{air}|}{A} = \rho_{air}gh_{air}$$
$$P_{tot} = P_{air} + P_{water}$$

Let's consider the relative magnitude of these two contributing pressures. While the column of air above the earth's surface is tens of kilometers thick, it is also about 1/1000 as dense as water. Air pressure is roughly constant, but does depend on both the altitude (affects h_{air}) and the particular weather conditions (affects ρ_{air}). For convenience, the pressure unit "one atmosphere" (abbreviated 1 atm) has been defined as 10^5 N/m^2 = 10^5 Pa. This is the air pressure which pushes inward on us all the time as we walk around. To get a sense of its magnitude, we can compare it to the water pressue at the bottom of a 1 m deep lake.

$$\begin{aligned} P_{water} &= \rho_{water} g h_{water} \\ &= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1\text{m}) \\ &\approx 10^4\text{Pa} \end{aligned}$$

This water pressure is 1/10 the pressure from the air, and we can computer the total pressure at the bottom of this 1-meter deep lake by using superposition:

$$P_{tot} = P_{water} + P_{air} = 10^4\text{Pa} + 10^5\text{Pa} = 1.1 \times 10^5\text{Pa}$$

Underwater Pressure Changes

A scuba diver descends by 15 meters, and on the way down she must "pop" her ears to equilibrate them with the changing pressure. What change in fluid pressure does she experience? At the upper location

$$P_i = \rho g h_i + 1\text{atm}$$

and at the lower location

$$P_f = \rho g h_f + 1\text{atm}.$$

Therefore, $\Delta P = \rho g \Delta h$. Notice that the atmospheric pressure has cancelled out during the subtraction to determine ΔP: it's only the change in depth that matters.

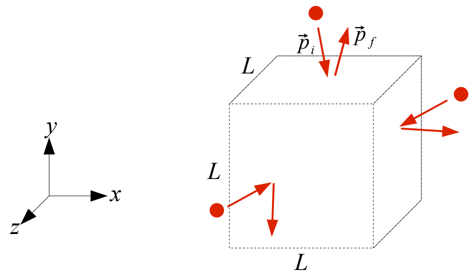
There is a practical consideration in this pressure change with depth. The gas in the diver's lungs needs to balance against this fluid pressure, so each breath the scuba diver breathes contains more molecules of gas to provide more collisions and hence more force. Therefore, she will use up her air tank at a faster rate the deeper she dives.

F.2.2 Microscopic origins of pressure

Read section 4.17 from *M&I: Modern Mechanics* (Vol. 1)

Pressure is a property that exists at any point in the fluid, not just at the bottom. What is providing this pressure? The collisions of the fluid molecules provide an impulse each time they bounce off an other objects: $\frac{\Delta \vec{p}}{\Delta t} = \vec{F}_{\text{net}}$ for each (of many) molecules.

Consider an imaginary cube of water (surrounded by other water which is no different from the water inside the cube), which is stationary.



As a molecule of the surrounding water bounces off the side of the box, it applies a force on the box. Of course, *many* collisions happen per second due to the large number of molecules. Since the box is stationary, we know from the Momentum Principle that $\vec{F}_{\text{net}} = \vec{F}_{molecules} + \vec{F}_{grav} = 0$ for all of these molecules plus the force due to Earth's gravity. We can also write $\vec{F}_{molecules} = \vec{F}_{left} + \vec{F}_{right} + \vec{F}_{front} + \vec{F}_{back} + \vec{F}_{top} + \vec{F}_{bottom}$ What can we say about the fluid forces on the left and right sides of the box? Front and back? Top and bottom?

From vector force balance, we know that $\vec{F}_{left} = -\vec{F}_{right}$ in the $\hat{x}$-direction and $\vec{F}_{front} = -\vec{F}_{back}$ in the $\hat{z}$-direction. However, this isn't true for the top and bottom surfaces, since we also have a gravitational force acting in the $-\hat{y}$-direction. Instead, $\vec{F}_{top} + \vec{F}_{bottom} + \vec{F}_{grav} = 0$ in order to explain why the box remains stationary. This is the direction which we need to examine carefully.

From what we know already learned about how pressure changes with depth,

$$P_{top} = \rho_{water} g h + 1\text{atm}$$

(acting on L^2 in the downward direction)

and

$$P_{bottom} = \rho_{water} g (h + L) + 1\text{atm}$$

(acting on L^2 in the upward direction)

where y is the distance between the top of the box and the surface of the water and P_{atm} is the atmospheric pressure. In order for the forces in the y-direction to balance

$$P_{top} L^2 (-\hat{y}) + P_{bottom} L^2 \hat{y} - m_{water} g \hat{y} = 0$$

and from our calculation of P_{top} and P_{bottom} we can write

$$\rho_{water} g L^3 = m_{water} g.$$

This makes sense, since $\rho_{water} L^3 = m_{water}$ is the usual definition of density, and we are thus confident that this force balance analysis make sense. To gain a deeper appreciation, look back through the chain of equations and notice that the m_{water} came from from the water *inside* the cube, and the ρL^3 came from the water *outside* the cube.

F.2.3 Archimedes Principle

Therefore, if we don't have a cube of water, but instead a cube made of a different material, we can nonetheless work through the exact same argument outlined above, but the forces will no longer balance since m_{water} will be replaced by m_{obj}. This allows us to write **Archimedes principle:**

the force from the surrounding fluid on a partially or fully submerged object is equal in magnitude to the weight of the fluid displaced. The net force on the object can be written

$$\vec{F}_{net} = \vec{F}_{fluid} + \vec{F}_{grav}$$

The direction of the net force comes from the relative densities of the object and the fluid: an object denser than the fluid will experience a downward force, while an object less dense than the fluid will experience and upward force. If the density ρ of the object were less than density of the surrounding fluid

$$F_{y,net} = -\rho_{obj} V g + \rho_{fluid} V g > 0$$

and therefore the object will experience a net upward force and start to rise. Similarly, if the density ρ of the object were greater than that of the surrounding fluid

$$F_{y,net} = -\rho_{obj} V g + \rho_{fluid} V g < 0$$

and the object will experience a net downward force and start to sink.

Microscopically, what is going on? If we return to the image of the cube with fluid molecules bouncing off its surfaces, we need to understand what is different about the collisions at the top and the bottom of the cube. It turns out that the *density* of the fluid molecules will be just slightly higher at the bottom, due to the increased pressure squeezing the molecules closer together. Therefore, there are slightly more molecules near the bottom surface (compared to the top) and thus more collisions per second and more transmitted force.

F.2.4 Archimedes Principle Example

Imagine holding an air-filled 2 L soda bottle ($V = 2000$ cm^3) underwater. You have used a scale to determine that the mass of the soda bottle is $m_{bottle} = 50$g. When you push it down to submerge it completely in a pool, what force is required to hold it underwater?

The submerged bottle displaces 2000 cm^3 of water, which has a density of 1 g/cm^3, and therefore a mass of 2 kg. From Archimedes principle, this "displaced water" provides an upward force of $m_{water} g = 19.6$ N upward on the bottle. In addition, the gravitational force on the bottle is $m_{bottle} g = 0.49$ N in the downward direction.

From the Momentum Principle for the static system consisting of only the bottle:

$$\frac{\Delta \vec{p}}{\Delta t} = \vec{F}_{net} = 0$$

$$\vec{F}_{net} = \vec{F}_{fluid} + \vec{F}_{grav} + \vec{F}_h and = 0$$

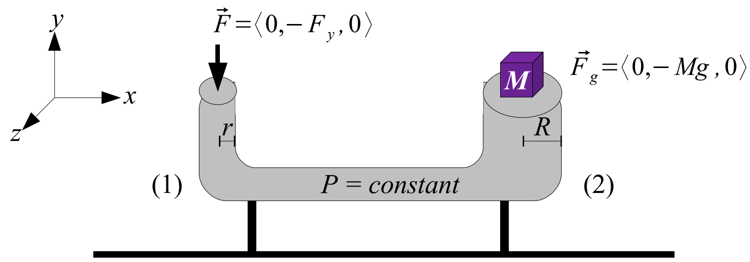
and we want to solve for the $\vec{F}_h and$ required to hold it stationary underwater.

$$\langle 0, 0, 19.6 \rangle N + \langle 0, 0, -0.49 \rangle N + \langle F_x, F_y, F_z \rangle = 0$$

Therefore, $F_x = F_y = 0$ and $F_z = (0.49 - 19.6)N = -19.1N$, which means we have to push downward with 19.1 N of force to keep the bottle submerged.

F.3 Hydraulics: Pascal's Principle

Because the molecules of a fluid are continually rearranging, every location within a fluid at the same height has the same pressure as any other. If this weren't the case, then the molecules in the higher-pressure areas would be pushed into the lower pressure areas and thus re-equilibrate the pressure. Therefore, consider the following system, in which a tube of fluid has openings of two difference sizes at its two ends.



You push down on end 1 (the narrow end) with a force $\vec{F}$ and want to raise the mass M which sits on end 2 (the wide end) at a constant, slow velocity. You will move slowly enough that the fluid can keep its pressure equilibrated at all times: it acts like a static fluid as described above. Because the mass is moving at a constant velocity, there is no change in momentum and $\Delta\vec{p}/\Delta t = 0$. We will use the Momentum Principle to solve for the force required, as a function of the other variables drawn in the figure above.

For a system composed of the fluid and the surroundings containing you (providing $\vec{F}$), the Earth, and the mass M, we know $\frac{\Delta\vec{p}_{fluid}}{\Delta t} = \vec{F}_{\text{net}} = 0$ for the fluid. We also have two different ways to calculate the pressure in the tube:

$$P = \frac{F_y}{A_1} = \frac{F_y}{\pi r^2}$$

$$P = \frac{Mg}{A_2} = \frac{Mg}{\pi R^2}$$

Since these two pressures must be equal (or else the fluid would rearrange to make it so),

$$\frac{F_y}{\pi r^2} = \frac{Mg}{\pi R^2}$$

and, solving for F_y,

$$F_y = Mg\left(\frac{r}{R}\right)^2.$$

Therefore, the smaller the ratio r/R, the less force you need to provide to move the mass M at constant velocity (or to hold it in place with $v = 0$). This is how hydraulic brakes and automobile repair shop lifts work.

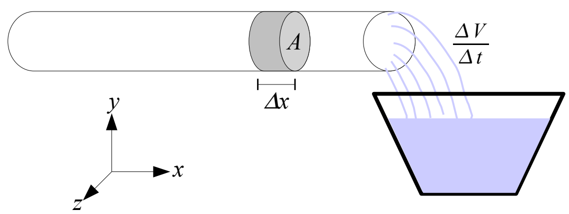
What is the microscopic explanation for why the force F_y is NOT just equal to Mg? End 2 has a larger area, so more molecules are colliding with it, providing a larger force (even though the pressure is the same at both ends).

F.4 Fluid Dynamics

Read section 12.2 from *M&I: Modern Mechanics* (Vol. 1)

F.4.1 Measuring Flow Rates

We will now shift our attention to fluids which are in motion, by first considering systems in which they move at constant velocity. Consider a tube with water filling up a bucket from an unseen source:



Zooming in on the molecules in the pipe, we can see that there are two types of motions going on. First, the molecules are bouncing off each other (and the walls of the pipe) due to their thermal motions. These velocities can be a few hundred m/s at any given instant, but in a (somewhat) random direction. However, the molecules are on average moving more to the right than to the left, since we see them flowing out of the pipe into the bucket, so we know that the direction of their individual velocities isn't completely random. On *average*, they have a larger component of their velocity in the $+\hat{x}$ direction than in the $-\hat{x}$ direction

At the macroscopic level, a segment of water with volume ΔV enters the pipe at the left, moves to the right and fills a bucket at a rate measured in volume per unit time. Here, we use Δ to mean a discrete piece of a particular variable, in the same sense that you have used it in calculus class.

$$\begin{aligned}
\Delta V &= \Delta x\,\Delta y\,\Delta z \\
&= A\Delta x \\
&= A\left(\frac{\Delta x}{\Delta t}\right)\Delta t \\
&= A\,v_{ave}\,\Delta t
\end{aligned}$$

Thus, we can use this expression to define a relationship between the average volume flow rate $\Delta V/\Delta t$ and the average velocity of the fluid molecules (in the same direction that the flow is happening):

$$\frac{\Delta V}{\Delta t} = A\,v_{ave}$$

Since volume flow rates are variously measured in liters/second or m^3/sec, it might be helpful to know that 1000 L = 1 m^3 = (10 cm)^3.

F.4.2 Calculating Flow Rates

For example, what is the average fluid velocity (in ft/sec) of the Haw River when it has a flow rate of 2000 ft^3/sec? The river depth was 6.5 feet at the time of this measurement, and the width of the Haw River at this particular geographic location is about 120 ft.

$$\frac{\Delta V}{\Delta t} = A\,v_{ave} = 2000\text{ft}^3/\text{s}$$

$$A = (6.5\text{ft}) \times (120\text{ft}) = 780\text{ft}^2$$

$$v_{ave} = 2.5\text{ft/s}$$

Note that if a boat were moving with the current (an approximation which doesn't correctly address the forces between the boat and the water), then this is how fast you would see the boat move downstream. If you wanted to know how much water was moving downstream, then the volume flow rate of 2000 ft^3/sec number would be more useful.

F.4.3 Energy Principle and Average Fluid Velocity

To show that v_{ave} really measures the average speed of the molecules in a meaningful way, consider pointing a garden hose directly upwards and measuring how high the fluid goes. This is a case where the Energy Principle is useful to describe the conversion of the fluid's kinetic energy to gravitational potential energy between the water and the Earth. Let's assume that the speed of the water is v_{ave} and is coming out of a tube pointed directly upwards. We will pick a segment of fluid of mass m and follow it upward, ignoring the pieces of fluid ahead or behind it.

system: segment of water + earth

$$\Delta E = \cancel{W_{ext}} + \cancel{Q}$$

$W, Q = 0$ since there is nothing external (neglecting friction) Only two energies (kinetic, potential) are changing in this situation, so:

$$\Delta K + \Delta U_{grav} = 0$$

which we can re-write

$$K_i + U_i = K_f + U_f$$

and substitute in our observed height, mass, and velocity:

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

$$\frac{1}{2}m(v_i^2 - v_f^2) = mg(h_f - h_i)$$

$$\frac{1}{2}\left(v_i^2 - v_f^2\right) = g(h_f - h_i)$$

For a reasonable initial velocity $v_i = 5$ m/s, and final position at the top of the trajectory ($v_f = 0$ m/s), we can solve for $\Delta h = h_f - h_i$.

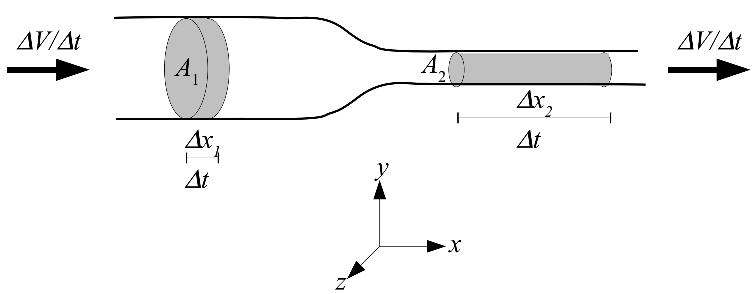
$$h_f - h_i = \frac{v_i^2}{2g} = 1.28m$$

If you wanted to measure how big of an effect friction was having, you could first find v_{ave} for a hose of known diameter by filling a bucket to measure $\Delta V/\Delta t$). Then, you could measure the height the water reaches when the hose points directly upwards, and compare to what you calculated above. How can you account for the over-estimation of height?

F.4.4 Continuity of Flow

Sometimes, you want to change the velocity of a fluid. In the garden hose example above, you might do this by putting your thumb partway over the opening. From experience, we know that the water will then exit the hose more quickly and you can thereby spray more distant parts of the garden.

We can generalize this situation to a pipe which changes diameter as the fluid flows through it.



We'll assume that the fluid is incompressible (e.g. a liquid, rather than a gas), so it can't just squeeze itself down to a smaller size. As the fluid moves from left to right, it is confined to a narrower space, but the same volume of fluid/sec needs to keep moving through the pipe. Therefore, the flow rate is constant and from our definition of flow rate we know that

$$\frac{\Delta V}{\Delta t} = A_1 v_1 \quad \text{and} \quad \frac{\Delta V}{\Delta t} = A_2 v_2$$

Therefore, the narrower pipe has to have a faster velocity in order to keep up with the flow rate, and

$$A_1 v_1 = A_2 v_2$$

for two portions of the same pipe. This is called a *continuity equation* for the fluid, and is related to conservation of mass.

If you examine a stream of water coming out of a faucet, you'll notice that as the stream moves downwards, it narrows. What does this tell you about the average velocity of the water molecules further down in the stream? Can you argue why this should be the case from the Energy Principle?

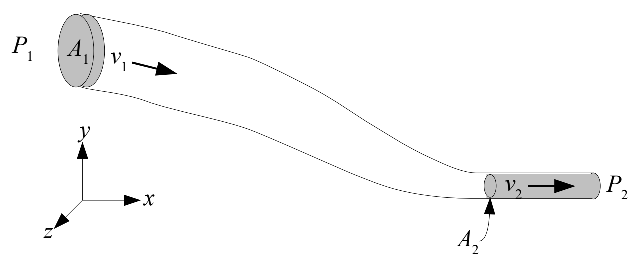
F.5 Bernoulli's Equation

What if we wanted to write $\Delta\vec{p} = \vec{F}_{net}\Delta t$ for a piece of moving fluid within a larger flow? Motion (momentum) affects pressure (in $\vec{F}_{net}$), but pressure also affects the motion! This means that the situation gets quite complicated. You can write the Momentum Principle for a fluid (called the Navier-Stokes Equation), but it's a nonlinear partial differential equation and best left for a specialized course on fluid dynamics.

However, it turns out that for a special case of problems, the Energy Principle can be informative and is significantly simpler to calculate. Imagine a ΔV of moving fluid within a long tube of changing diameter, which moves from one point to another within the flow (left to right), it's being pushed from the left by one pressure, and pushed from the right by another, smaller, pressure. In both cases, the pressure provides a force acting over a distance. For problems that looks something like this (even if the "tube" is imaginary: called a *streamline*), we can use Bernoulli's Equation, which is a fluid-specific interpretation of the Energy Principle.

F.5.1 Derivation of Bernoulli's Equation

Consider a particular segment of fluid moving along an imaginary tube. We'll make an unusual choice of system: the chosen segment of fluid, with the rest of the fluid and the Earth making up the surroundings. We'll follow the segment of fluid as it moves from left to right, and perhaps changing its diameter, moving vertically, and changing velocity all at the same time. The fluid segment motion is caused by a pressure P_1 pushing on the tube from the left side, a pressure P_2 pushing on the tube from the right side (due to the surrounding fluid), as well as gravitational forces.



At the left end of a tube of flow, the segment of fluid moves though a distance Δx_i, and the force pushing the fluid forward is $F_i = P_i A_i$. Since the displacement and force point in the same direction, $W_i = P_i A_i \Delta x_i$ is positive. Notice that the last two factors are just ΔV (the volume of the segment of fluid). At the other end of the tube, the same ΔV is pushed out (due to its incompressibility). The work $W_f = -P_f A_f \Delta x_f = -P_f \Delta V$ is negative since the pressure and displacement point in opposite directions.

Second, we have a gravitational force acting on the segment as it is displaced, which does work on the fluid segment:

$$\begin{aligned} W_g &= \vec{F}_g \cdot \Delta\vec{r} \\ &= -mg(h_f - h_i) \\ &= -\rho\Delta V g(h_f - h_i) \end{aligned}$$

where the mass of the fluid is $m = \rho\Delta V$. Therefore, we have a net external force due to both fluid pressure and gravity

$$W_{ext} = W_{P_i} + W_{P_f} + W_g$$

We can write the Energy Principle for this segment of fluid, where the only energy that is changing is the kinetic energy:

$$\Delta E = W_{ext} + \cancel{Q}$$

$$\Delta K = W_{ext}$$

$$\frac{1}{2}(\rho \Delta V)(v_f^2 - v_i^2) = P_i \Delta V - P_f \Delta V - (\rho \Delta V) g (h_f - h_i)$$

We can divide through by ΔV: the physical interpretation is that it doesn't matter what particular small segment of fluid you choose. Rearranging terms, this gives us Bernoulli's Equation:

$$\frac{1}{2}\rho v_f^2 + \rho g h_f + P_f = \frac{1}{2}\rho v_i^2 + \rho g h_i + P_i$$

or

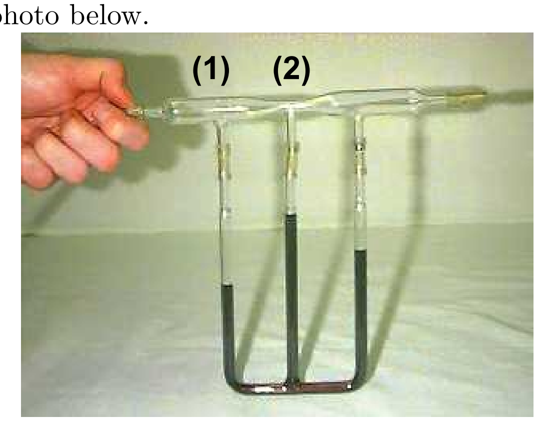
$$\frac{1}{2}\rho v^2 + \rho g h + P = constant$$

As you compare two points along the flow, it will always be true that $\Delta(KE/volume) + \Delta(U/volume) + \Delta P = 0$. If the segment of fluid experiences and increase its kinetic energy, that energy change came from a decrease in one of the two other terms. It may be helpful to recall the microscopic description of the fluid pressure: the molecules in the fluid collide with the surrounding fluid and transfer linear momentum. Part of the momentum of the molecules is in a random direction (producing P), but a portion will also be in the direction of the flow (v_{ave}), and will instead contribute to the kinetic energy term.

F.6 Applications of the Bernoulli Equation

F.6.1 Venturi Flow Meter

The Bernoulli Equation provides a convenient way to measure the fluid flow rate at different points along a flow, using a device known as a Venturi flow meter. Consider a pipe which changes its diameter from a wide region (1) to narrow (2) and then back to a wide region again, as shown in the horizontal pipe in the photo below.



The flow meter is the fork-like set of tubes connected to the bottom. The dark fluid can flow freely within the flow meter, and is observed to have risen to the largest height within the middle tube (connected to the narrowest part of the flow.) We will use the Bernoulli Equation to explain this observation.

Due to the continuity of the fluid (see §F.4.4), the decrease in cross-sectional area requires a corresponding increase in the fluid velocity through that constriction:

$$v_1 = v_2 \left(\frac{A_2}{A_1} \right).$$

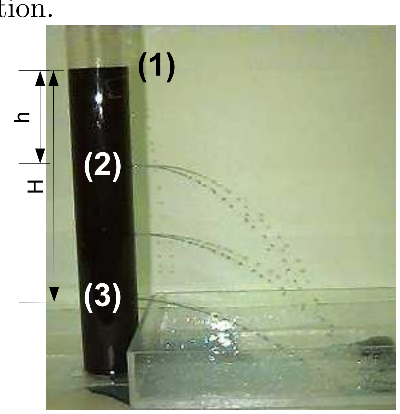
Since $A_1 < A_2$, we know that $v_1 < v_2$. When the pipe is kept at constant $h_1 = h_2$,

$$\frac{1}{2}\rho v_1^2 + \cancel{\rho g h_1} + P_1 = \frac{1}{2}\rho v_2^2 + \cancel{\rho g h_2} + P_2.$$

and from the difference in velocities we can infer that $P_1 > P_2$. Note that for the location with the larger velocity, the lower pressure manifests itself by its (relative) inability to push the blue flow down. (Microscopically, recall that pressure comes from collisions: lower pressure means fewer collisions per area.)

F.6.2 Outflow from a Tank

Imagine poking a series of holes in the side of a 2 liter soda bottle: streams of water would come out of each hole. How would the speed of the streams depend on where the hole was located? The photo below shows such a situation.



Which stream is leaving the cylinder fastest? Why? What is the evidence from the photo?

We will use Bernoulli's Equation to determine the outflow velocity at each hole. Consider the three locations marked on the picture. At location (1), at the top of the water column,

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1 = 0 + 0 + P_{atm} \equiv B.$$

We have assigned this expression the value B so that we can use it when making comparisons to similar calculations further down the column. At locations (2) and (3), respectively,

$$\frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2 = \frac{1}{2}\rho v_2^2 - \rho g h + P_{atm}$$

and

$$\frac{1}{2}\rho v_3^2 + \rho g h_3 + P_3 = \frac{1}{2}\rho v_3^2 - \rho g H + P_{atm}.$$

We also know that a streamline connects (1) to (2) and a different streamline connects (1) to (3). Therefore, we can write two Bernoulli Equations, one for each streamline:

$$B = \frac{1}{2}\rho v_2^2 - \rho g h + P_{atm}$$

and

$$B = \frac{1}{2}\rho v_3^2 - \rho g H + P_{atm}$$

By setting the right hand sides equal to each other, we can eliminate B and write:

$$\frac{1}{2}\rho v_2^2 - \rho g h + P_{atm} = \frac{1}{2}\rho v_3^2 - \rho g H + P_a tm$$

Which reduces to

$$v_2^2 - v_3^2 = 2g(h - H)$$

and tells us that since $h < H$, both sides of the equation are negative and therefore $v_2 < v_3$.

As the fluid drains from the cylinder over time, what will happen to the velocity measured at location (3)? Will it increase, decrease, or remain the same?

F.7 Learning Objectives

After studying the material in this chapter, you should be able to:

- relate the microscopic and macroscopic descriptions of fluid pressure
- explain Archimedes' Principle in terms of changes in fluid pressure with depth
- calculate the buoyant force on an object immersed in a fluid
- use pressure as a contribution to calculations of $\vec{F}_{net}$
- relate the microscopic and macroscopic descriptions of fluid velocity
- quantitatively relate a change in fluid velocity to a change in pipe cross section through continuity
- relate changes in pressure, velocity, and height through the Energy Principle (Bernoulli's Equation)