Matter & Interactions Fluids Unit

Karen Daniels

8 February 2009

Contents

1 Note to Instructors

1.1 Learning Objectives

- relate micro and macro origins of fluid pressure
- explain Archimedes' Principle in terms of changes in fluid pressure with depth (uses momentum principle for static equilibrium)
- quantitatively relate microscope average velocity to macroscopic flow rate quantitatively relate a change in fluid velocity to a change in pipe cross section (continuity)
- state whether a given velocity vector field (picture) contains a source, sink, or vorticity
- relate changes in pressure, velocity, and height through the Energy Principle (Bernoulli's Equation)

1.2 Connections to other parts of the M&I sequence

- momentum, energy principles
- cross-sectional areas (connects to Young's modulus calculations in 1st semester,
- micro/macro current in 2nd semester)
- pressure (like Young's modulus ball & spring, each internal point is in balance)
- another place to see diverging vs. curly vector fields

1.3 Readings

- Section 4.17 from M&I textbook
- Chapter 33 & 34 from Calculus-Based Physics (A Online Free Physics Textbook by Jeffrey W. Schnick):

  http://www.anselm.edu/internet/physics/cbphysics/downloadsI/cbPhysicsIa17.pdf

1.4 WebAssign

- HW #1: Assignment ID 612645
- HW #2: Assignment ID 612652 (Need to check: I think this had the Bernoulli Equation question removed from it ...)
- Lab: Assignment ID 612649

2 What is a Fluid?

So far, we've mostly talked about solid objects, but the same fundamental principles (momentum, energy) can also tell us about fluids. Will make use of both microscopic and macroscopic reasoning, just as for the ball & spring model of a solid.

2.1 Brainstorm solid vs. fluid

Solids

- can hold own shape
- atoms keep same neigbors, can be crystalline
- close molecules
- higher density

Fluids

- takes shape of container (or surface tension)
- molecules diffuse around, not crystalline structure
- molecules farther apart
- lower density (generally)

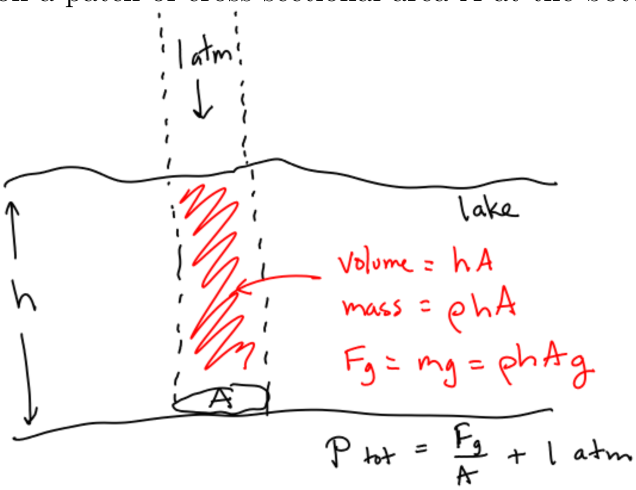
Microscopic definition of a fluid: multi-particle system of random (but interacting) motion of cohesive particles (atoms/molecules) Macroscopic definition: takes shape from container or surface tension

To disambiguate gases and liquids: gasses are compressible, while fluids are not very compressible. Gases will expand to fill a closed container, while fluids will only settle to the bottom. The distinction between the last two results from a consideration of whether the thermal energy is enough to destroy all connection to neighboring molecules/atoms.

We'll model fluids both as a macroscopic object (a total mass at a particular location) and as a collection of molecules, just as we did for the ball and spring model of a solid. The difference is that there are now many collisional balls bouncing around in a box, rather than being connected by spring-like forces. This explains why fluids take the shape of their container and have diffusion.

3 Fluid Pressure

Consider a patch of ground at the bottom of a lake. What is the force from the water and the air on a patch of cross-sectional area A at the bottom?



3.1 Macro picture

A mass of water is sitting on the lake bottom, and a mass of air is sitting on top of the water. Calculate the mass of each:

$$F_{water} = m_{water}g = (\rho_{water}V)g = (\rho_{water}Ah_{water})g$$

$$F_{air} = m_{air}g = (\rho_{air}V)g = (\rho_{air}Ah_{air})g$$

We can convert this to pressures: Force / Area

$$P_{water} = \frac{F_{water}}{A} = \rho_{water}gh_{water}$$

$$P_{air} = \frac{F_{air}}{A} = \rho_{air}gh_{water}$$

The column of air pushing down on our patch, due to gravity, has a height of 10s of kilometers, and it is much less dense than water. Let's compare the pressure due to the water (for a lake with a depth of 1 m) to the atmospheric pressure. Can measure the atmospheric pressure of air (1 atm), and it turns out to be about 10^5 Pa [N/m^2], although it does depend on altitude and weather conditions.

$$\begin{aligned} P_{water} &= \rho_{water}gh_{water} \\ &= \left(\frac{1000kg}{m^3}\right)\left(\frac{9.8m}{s^2}\right)(1m) \\ &\approx 10^4 Pa \end{aligned}$$

This is $10\times$ less than the pressure from the air! What's the total pressure at the bottom of this 1-meter deep lake? Use superposition.

$$P_{tot} = P_{water} + Pair = 10^4 Pa + 10^5 Pa = 1.1 \times 10^5 Pa$$

Clicker Question: Scuba Diver

A scuba diver descends from a depth of 10 meters to a depth of 25 meters. What is the *change* in pressure between these depths?

1. 1×10^5 Pa
2. 1.5×10^5 Pa $\Longleftarrow$
3. 2×10^5 Pa
4. 2.5×10^5 Pa

When going over problem, note that atmospheric pressure cancels out, and it's only the change in depth that matters: $\Delta P = \rho g \Delta h$

3.2 Micro picture

Pressure is a property that exists anywhere in the fluid. What property of the fluid is providing this pressure? The collisions of the molecules provide an impulse on another object: $\frac{\Delta \vec{p}}{\Delta t} = \vec{F}_{\text{net}}$ for each (of many) molecules.

Think carefully about the following question. [Draw picture of box to help, sides of length L]

As each molecule boundes off the side of the box, it applies this $\Delta\vec{F}$ to the box, and many collisions happen per second due to the large number of molecules.

What does the momentum principle say about the forces? $\vec{F}_{\text{net}} = 0$

Clicker Question: Pressure Balance

Consider a scuba diver who has taken a cubical box down to a depth of 10m. We observe that the box is able to sit in the middle of the water without moving. Therefore, $\Delta\vec{p}_{\text{box}} = 0$ Consider the momentum principle, and determine which of the following pairs of forces balance each other?

A. net fluid force and force of gravity	1. A only
B. fluid force on the left side and fluid force on the right side	2. A, B, and C $\Longleftarrow$
C. fluid force on the front side and fluid force on the back side	3. A, B, C, and D
D. fluid force on the top side and fluid force on the bottom side	4. B, C, and D

Answers 2 & 3 are both popular. Use symmetry to talk about left/right and front/back collisions being equivalent (same force, same area, same pressure).

Micro picture: think about this in terms of the momentum principle. Where does this pressure (force) come from? Why would the fluid force be different on the top and bottom sides?

$$\langle P_{top} + P_{bot}, P_{left} + P_{right}, P_{front} + P_{back} \rangle A = \langle 0, 0, 0 \rangle$$

Left/right must balance, front/back must balance, top/bottom must balance.

Look more carefully at top/bottom pair

Force from fluid at $P(z = top) = \rho_{water} gzA$ in downward direction Force from fluid at $P(z = top + L) = \rho_{water} g(z + L)A$ in upward direction

$F_z = \rho_{water} gLA = m_{water} g$ upward

Archimedes principle: the fluid force on a partially or fully submerged object is equal in magnitude ot the weight of the fluid displaced. The direction of the force comes from the relative densities of the object and the fluid (floating vs. sinking.)

If density of box is less than density of water: $F_{z,net} = -\rho_{box} V g + \rho_{fluid} V g > 0$, so box will rise If greater density $\rightarrow$ sink

What causes this? Draw molcules bouncing off bottom and top surfaces. What's different at the top and bottom of the box? Turns out that the *density* of water molecules is just slightly higher at the bottom, so there are more collisons/second and therefore the force is higher. Same would be true for a "ghost box" composed of water: more collisions from below than from above, so the pressure is somewhat higher at the bottoms. [Draw ghost box, with dotted lines, for comparison.]

3.3 Demo: Floating Coke Cans (#2B40.291)

4 Applications of Fluid Pressure

4.1 Lab: Archimedes Principle

For Scale-Up (if there's enough time): Instruct class to load lab from WebAssign and read the instructions. Before beginning data collection, the whole table must draw (on the wall whiteboards) a prediction for what the graph will look like.

Otherwise, we'll walk through the lab as an abbreviated demo in class? This might be a good place to have them practice writing explanations of the observed graph (as plotted on the screen).

4.2 Whiteboard/Worked problem: Iceberg Density

[Done by instructor if using lecture format.]

It is conventional to say that 90% of an iceberg lies below the sea surface. If this were the case, what would be the density of an iceberg? Does the shape of the iceberg matter to your answer?

Start from a fundamental principle: observe that $\frac{\Delta \vec{p}_{\text{iceberg}}}{\Delta t} = \vec{F}_{\text{net}} = 0$

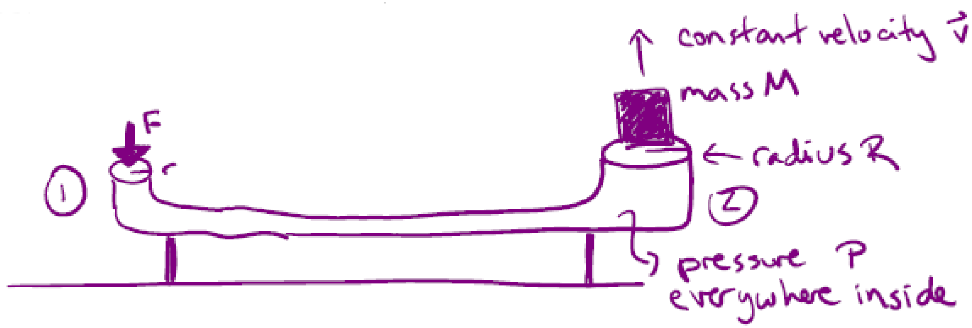
What are the net forces? $F_{\text{net}} = F_{\text{grav}} + F_{\text{bouy}}$ and these are only in the z-direction.

Call mass of iceberg M, mass of displaced water m, substitute ρV for each, using $0.9V$ for the displaced volume. Solve for $\rho_{\text{iceberg}} = 0.9$ g/cm^3

[Compare this to an actual value found online.]

4.3 Whiteboard/Worked problem: Pascal's Principle/Hydraulics [Optional]

Consider the following system:



You push down on side 1 (the skinny side) with a force $\vec{F}$ and want to raise the mass M which sits on side 2 (the fat side) at a constant slow velocity. Static fluids are able to equilibrate pressure everywhere within the fluid. Use the momentum principle to solve for the value of the force F required to lift the mass M, in terms of such variables as $(M, r, R, v, g,$ etc.) Be sure to specify an appropriate choice of system and surroundings.

system = fluid
$\frac{\Delta \vec{p}_{\text{iceberg}}}{\Delta t} = \vec{F}_{\text{net}} = 0$ for fluid
Therefore $|F_1| = |F_2|$ and also know that $P = const.$ within horizontal fluid.

$$P = \frac{F_1}{A_1} = \frac{F_1}{\pi r^2}$$

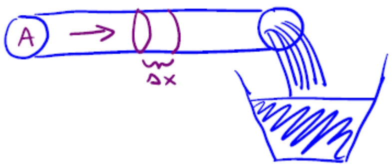
$$P = \frac{F_2}{A_2} = \frac{Mg}{\pi R^2}$$

$$F_1 = Mg\frac{r^2}{R^2}$$

What is the microscopic explanation for why the force F_1 is NOT equal to Mg? Side 2 has a larger area, so more molecules are colliding with it, providing a larger force (even though the pressure is the same at both ends).

5 Fluid Velocity

Consider a tube with water flowing through it:



Micro: How fast are the water molecules moving? Want to consider two type of motions, the bouncing around due to thermal motions, and the average flow velocity. Particles bouncing around (a few 100 m/s at any given instant, but somewhat random direction). On AVERAGE, they have some speed, since you can put a bucket under the stream and fill it up.

Macro: A "chunk" of water with volume ΔV enters the pipe at the left and moves to the right and fills a bucket at a rate of liters/second or m^3/sec (volume/time).

$$\begin{aligned}\Delta V &= \Delta x\,\Delta y\,\Delta z \\ &= A\Delta x \\ &= A\frac{\Delta x}{\Delta t}\Delta t \\ &= A v_{ave}\Delta t\end{aligned}$$

Define flow rate:

$$\left(\frac{\Delta V}{\Delta t}\right) = A v_{ave}$$

relates flow rate (liters/sec) to fluid velocity (m/s)

1000 L = 1 m^3 = (10 cm)^3

Clicker Question: Flow Rate

What's the flow rate (in m^3/s) of a garden hose which fills 1 L of a bucket every 10 seconds?

1. 1e-1 m^3/s
2. 1e-2 m^3/s
3. 1e-3 m^3/s
4. 1e-4 m^3/s $\Longleftarrow$
5. 1e-5 m^3/s

Clicker Question: Fluid Velocity

What is the fluid velocity of the water stream with a flow rate of 1×10^-4 m^3/s, through a 2 cm garden hose?

1. 4 m/s
2. 12.6 m/s $\Longleftarrow$
3. 1.2 e-7 m/s
4. 0.08 m/s (reciprocal of correct answer)

5.1 Whiteboard/Worked Problem: Aiming Water Hose Upwards

To show that v_{ave} really measures the average speed of the molecules, consider pointing a garden hose upwards. How high does the fluid go?

If the speed of the water is 5 m/s and the diameter of the hose is 2 cm, do the calculation using the energy principle.

Pick a little piece of fluid of mass mand follow it (ignore the other pieces of fluid)

Choose system = bit of water + earth

$$E_f = E_i + W + Q$$

$W, Q = 0$ since nothing external

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

$$\frac{1}{2}m(v_i^2 - v_f^2) = mg(h_f - h_i)$$

$$\frac{1}{2}v_i^2 - v_f^2 = g(h_f - h_i)$$

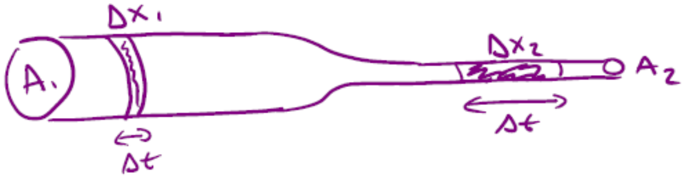
$v_i = 5$ m/s, $v_f = 0$ m/s, $h_i = 0$, solve for h_f

$$h_f = v_i^2\ over 2g = 1.28m$$

We neglected friction here. If you wanted to estimate how big of an affect it was, you could measure v by filling bucket (to get $\Delta V/\Delta t$) and then measure the height the water reaches when the hose points directly upwards.

5.2 Contintuity of Flow

What if the tube changes diameter? What happens to the speed of the fluid? We'll assume that it's incompressible, so it can't just squeeze down to a smaller size.



Clicker Question: Continuity Concept

A wide pipe is connected to a narrow pipe and water flows in it with a constant flow rate. Compared to the fluid velocity in the wide pipe, the fluid velocity in the narrow pipe is

1. faster $\Longleftarrow$
2. slower
3. same speed
4. not enough information to tell

Flow rate $\frac{\Delta V}{\Delta t}$ is constant. So $A_1 v_1 = A_2 v_2$, and therefore the skinnier pipe has to have a faster velocity in order to keep up with the flow rate. This is called *continuity* of the fluid: related to mass conservation.

Clicker Question: Continuity Calculation

A wide pipe is connected to a narrow pipe with half the radius and water flows in it with a constant flow rate. If the fluid velocity in the wide pipe is v, the fluid velocity in the narrow pipe is Whiteboard Problem

1. $v/4$
2. $v/2$
3. v
4. $2v$
5. $4v$ $\Longleftarrow$

This is where we ended in Scale-Up last year ... anything past this point is less well-developed.

---

6 Bernoulli's Equation

What if we wanted to write $\Delta\vec{p} = \vec{F}_{net}\Delta t$ for a piece of moving fluid within a larger flow? Motion (momentum) effects pressure (in F_{net}), but pressure also effects the motion! This means that the situation gets quite complicated! You can write the momentum equation for a fluid, and it's called the Navier-Stokes Equation. Can take a fluids class to learn more about that.

It turns out that for a special case of problems, the Energy Principle is easier to use. Imagine a ΔV of moving fluid within a long tube of changing diameter, which moves from one point to another within the flow (left to right), it's being pushed from the left by one pressure, and pushed from the right by another: this is a force acting over a distance. For problems that looks something like this (even if the "tube" is imaginary), we can use Bernoulli's Equation, which is a fluids version of the Energy Principle.

6.1 Derivation of Bernoull's Equation

Consider a tube of fluid (the system is this bit of fluid plus the Earth) moving from left to right, and perhaps changing its diameter as it goes. There's a pressure P_1 pushing on the tube from the left side, and a pressure P_2 pushing on the tube from the right side.

At the left end of a tube of flow, the chunk of fluid moves though a distance Δx_1, and the force pushing the fluid forward is $F_1 = P_1A_1$. Do the displacement and force point in the same or opposite direction? Same, so $W_1 = P_1A_1\Delta x_1$. Notice that the last two factors are just ΔV (the volume of the chunk of fluid).

What about at the other end? The same ΔV is pushed out (incompressibility). What is the work? $W_2 = -P_2A_2\Delta x_2 = -P_2\Delta V$. Pressure and displacement now point in OPPOSITE directions, since outgoing fluid has to push fluid out of the way to move forward, so the sign is negative.

Therefore, we have $W_{net} = W_i + W_f = -(P_f P_i)\Delta V$ and we can write the energy principle for this chunk of fluid:

$$\Delta K + \Delta U = W_{ext}$$
$$\frac{1}{2}(\rho\Delta V)(v_f^2 v_i^2) + (\rho\Delta V)g(z_f z_i) = -(P_f - P_i)\Delta V$$

We can divide through by ΔV (the physical interpretation is that it doesn't matter what particular small chunk of fluid you choose) This gives us Bernoulli's Equation:

$$\frac{1}{2}\rho v_f^2 + \rho g z_f + P_f = \frac{1}{2}\rho v_i^2 + \rho g z_+ P_i = constant$$

As you compare two points in the flow, it will always be true that $\Delta(KE/volume) + \Delta(U/volume) + \Delta(P) = 0$.

7 Applications of BE

7.1 Demo: Venturi Meter (#2C20.26)

Change in area → change in velocity → change in pressure

Micro explanation? More pressure = more collisions/area Recall that pressure comes from collisions: molecules push the next set of molecules along.

7.2 Demo/Whiteboard/Worked Problem

Velocity of Efflux - Three Hole Can Experiment (Demo #2C10.10)

Why does water come out of bottom hole faster?

Brainstrom Observations as Class:

Whiteboard: Calculate Velocity

8 Sources & Sinks

Consider bulk flows of fluids. Show diverging and curly velocity vector fields (might be useful for E&M). Talk about sources, sinks, vorticity. How do you see continuity on these images

Clicker Question: Velocity Fields

Which velocity field could represent the outflow of an incompressible fluid from a hole in the center of a region?